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INTUITIONISTIC FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM (IFMOLPP) USING TAYLOR SERIES APPROACH

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Index Terms— Intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP), membership and non-membership functions, Taylor series.

1 INTRODUCTION

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. The concept of Fuzzy sets was introduced by Zadeh in 1965. In the literature, different approaches appear to solve different models of linear programming problem (LPP).Because, programming solves more efficiently the above problems with respect to the some constraints. The concept of maximizing decision was initially proposed by Bellman and Zadeh [3]. By adopting this concept of fuzzy sets was applied in mathematical programs firstly by Zimmerman [20]. In the past few years, many researchers have come to the realization that a variety of real world problems which have been previously solved by linear programming techniques are in fact more complicated. Frequently, these problems have multiple goals to be optimized rather than a single objective. Moreover, many practical problems cannot be represented by linear programming model.

Therefore, attempts were made to develop more general mathematical programming methods and many significant advances have been made in the area of multi-objective linear programming. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) [1,2] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives.

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Certainly fuzzy sets theory is not appropriate to deal with such problems; rather intuitionistic fuzzy sets (IFS) theory is more suitable. The Intuitionistic fuzzy set was introduced by Atanassov.K.T [1] in 1986.

For the fuzzy multiple criteria decision making problems, the degree of satisfiability and non- satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number (IFN), which is an element of IFS [8,16].

This Intuitionistic fuzzy mathematics is very little studied subject. In a recent review, Toksari [5] gave a Taylor series approach to fuzzy multi-objective fractional programming problem. However, many methods of solving multi-objective linear programming problems are available in the literature. In this paper, membership and non-membership functions, which are associated with each objective of intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP) are transformed by using first order Taylor polynomial series [5]. Then the IFMOLPP can be reduced to a single objective linear programming. The paper is organized as follows:

The formulation of the problem is given in Section 2, and Section 3 deals with an algorithm for solving a intuitionistic fuzzy multi-objective linear programming problem. Finally, in Section 4, the effectiveness of the proposed method is illustrated by means of an example. Some concluding remarks are given in section 5.

2. FORMATION OF THE PROBLEM

The multi-objective linear programming problem and the multi- objective fuzzy linear programming problem are described in this section.

2.1. Multi-objective linear programming problem (MOLPP)

A linear multi-objective optimization problem is stated as Maximize or Minimize: $[z_1(x), z_2(x), \ldots\ldots, z_k(x)]$

Subject to $Ax \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b$, $x \ge 0$
where $z_j(x)$ j = 1, 2,… n is an N vector of cost coefficients, A an m x N – Coefficients matrix of constraints and b an m vector of demand (resource) availability.

2.2. Intuitionistic fuzzy multi-objectivelinear programming problem (IFMOLPP)

If an imprecise aspiration level is introduced to each of the objectives of MOLPP, then these fuzzy objectives are termed as fuzzy goals.
Let g_k^I be the aspiration level assigned to the kth objective $z_k(x)$.Then the fuzzy objectives appear as

(i) $Z_k(x) \gtrsim g_k^I$ (for maximizing $Z_k(x)$);
(ii) $Z_k(x) \lesssim g_k^I$ (for minimizing $Z_k(x)$);

Where $\gtrsim$ and $\lesssim$ indicate the fuzziness of the aspiration levels, and is to be understood as "essentially more than" and "essentially less than in the sense of Zimmerman[18]. Hence, the intuitionistic fuzzy multi-objective linear programming problem can be stated as follows:

Find X
so as to satisfy $Z_i(x) \lesssim g_i^I$, i=1,2,...............i_l,
$Z_i(x) \gtrsim g_i^I$, i=i_{l+1}, i_{l+2},……………,k
Subject to $x \in X$, $X \ge 0$.

Now, in the field of intuitionistic fuzzy programming, the intuitionistic fuzzy objectives are characterized by their associated membership functions and non-membership functions [10]. They can be expressed as follows:

If $Z_i(x) \gtrsim g_i^I$, $\mu_i^I(x) = \begin{cases} 1, & \text{if } z_i(x) \ge g_i \\ \frac{z_i(x)-\underline{t_i}}{g_i-\underline{t_i}}, & \text{if } \underline{t_i} \le z_i(x) \le g_i \\ 0, & \text{if } z_i(x) \le \underline{t_i} \end{cases}$ and

$$\nu_i^I(x) = \begin{cases} 0, & \text{if } z_i(x) \ge g_i \\ \frac{z_i(x)-g_i}{\underline{t_i}-g_i}, & \text{if } \underline{t_i} \le z_i(x) \le g_i \\ 1, & \text{if } z_i(x) \le \underline{t_i} \end{cases}$$

If $Z_i(x) \lesssim g_i^I$, $\mu_i^I(x) = \begin{cases} 1, & \text{if } z_i(x) \le g_i \\ \frac{\overline{t_i}-z_i(x)}{\overline{t_i}-g_i}, & \text{if } g_i \le z_i(x) \le \overline{t_i} \\ 0, & \text{if } z_i(x) \le \overline{t_i} \end{cases}$ and

$$\nu_i^I(x) = \begin{cases} 0, & \text{if } z_i(x) \ge g_i \\ \frac{g_i-z_i(x)}{\underline{t_i}-g_i}, & \text{if } \underline{t_i} \le z_i(x) \le g_i \\ 1, & \text{if } z_i(x) \le \overline{t_i} \end{cases}$$

where $\overline{t_i}$ and $\underline{t_i}$ are the upper tolerance limit and the lower tolerance limit respectively, for the ith intuitionistic fuzzy objective.

Now, in a intuitionistic fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible are actually represented by the possible achievement of their respective membership values and non-membership values to the highest degree. The relationship between constraints and the objective functions in the intuitionistic fuzzy environment is fully symmetric, that is, there is no longer a difference between the former and the latter [15]. This guarantees the maximization of both objectives' membership values and non-membership values simultaneously.

3. ALGORITHM FOR INTUITIONISTIC FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM

According to Toksari [5],in the intuitionistic fuzzy multi-objective linear programming problem, membership functions and non-membership functions associated with each objective are transformed by using Taylor series at first and then a satisfactory value(s) for the variable(s) of the model is obtained by solving the fuzzy model, which has a single objective function. Based on this idea, an algorithm for solving intuitionistic fuzzy multi-objective linear programming problem is developed here.

Step 1.Determine $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots\ldots\ldots, x_{in}^*)$, that is used to maximize or minimize the ith membership function $\mu_i^I(x)$ and non-membership function $\nu_i^I(x)$ (i=1,2,……….k) where n is the number of variables.

Step 2.Transform membership and non-membership functions by using first-order Taylor polynomial series

$$\mu_i^I(x) \cong \widehat{\mu_i^I}(x) = \mu_i^I(x_i^*) + [(x_1 - x_{i1}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_1} + (x_2 - x_{i2}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_2} + \ldots\ldots + (x_n - x_{in}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_n}]$$

$$\mu_i^I(x) \cong \widehat{\mu_i^I}(x) = \mu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_j}$$

$$\nu_i^I(x) \cong \widehat{\nu_i^I}(x) = \nu_i^I(x_i^*) + [(x_1 - x_{i1}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_1} + (x_2 - x_{i2}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_2} + \ldots\ldots + (x_n - x_{in}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_n}]$$

$\nu_i^I(x) \cong \widehat{\nu_i^I}(x) = \nu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_j}$

Step 3.Find satisfactory $x^* = (x_1^*, x_2^*, \dots\dots\dots, x_n^*)$ by solving the reduced problem to a single objective for membership function and non-membership function respectively.

$p(x) = \sum_{i=1}^{k} \mu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_j}$

and $q(x) = \sum_{i=1}^{k} \nu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_j}$.

Thus FMOLPP is converted into a new mathematical model. The model is as follows:

Maximize or Minimize $\sum_{i=1}^{k} \mu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \mu_i(x_i^*)}{\partial x_j}$

and

Maximize or Minimize $\sum_{i=1}^{k} \nu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \nu_i^I(x_i^*)}{\partial x_j}$

where $\mu_i^I(x) = \begin{cases} 1, & \text{if } z_i(x) \geq g_i \\ \frac{z_i(x) - \underline{t_i}}{g_i - \underline{t_i}}, & \text{if } \underline{t_i} \leq z_i(x) \leq g_i \\ 0, & \text{if } z_i(x) \leq \underline{t_i} \end{cases}$ and

$$\nu_i^I(x) = \begin{cases} 0, & \text{if } z_i(x) \geq g_i \\ \frac{z_i(x) - g_i}{\underline{t_i} - g_i}, & \text{if } \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & \text{if } z_i(x) \leq \underline{t_i} \end{cases}$$

and $\mu_i^I(x) = \begin{cases} 1, & \text{if } z_i(x) \leq g_i \\ \frac{\overline{t_i} - z_i(x)}{\overline{t_i} - g_i}, & \text{if } g_i \leq z_i(x) \leq \overline{t_i} \\ 0, & \text{if } z_i(x) \leq \overline{t_i} \end{cases}$ and

$$\nu_i^I(x) = \begin{cases} 0, & \text{if } z_i(x) \geq g_i \\ \frac{g_i - z_i(x)}{\underline{t_i} - g_i}, & \text{if } \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & \text{if } z_i(x) \leq \overline{t_i} \end{cases}$$

respectively.

4. Illustrative Example

Consider the following MOLPP:

$$\left.\begin{aligned} &\text{Maximize } z_1(x) = x_1 + x_2 \\ &\text{Maximize } z_2(x) = 3x_1 - 2x_2 \\ &\text{Subject to the constraints } 2x_1 + x_2 \leq 6 \\ &\qquad x_1 + 4x_2 \leq 12 \end{aligned}\right\} \quad (4.1)$$

a) The membership and non-membership functions were considered to be intuitionistic triangular (see figure 1) when they depend on three scalar parameters (a_1, b_1, c_1). z_1 depends on three scalar parameters (2,3.5,5) when z_2 depends on three scalar parameters (-0.01,9,18). The membership and non-membership functions of the goals are as obtained as follows: $\mu_1^I(x) =$

$$\begin{cases} 0, & \text{if } z_1(x) \geq c_1 \\ \frac{c_1 - z_1(x)}{c_1 - b_1}, & \text{if } b_1 \leq z_1(x) \leq c_1 \\ \frac{z_1(x) - a_1}{b_1 - a_1}, & \text{if } a_1 \leq z_1(x) \leq b_1 \\ 0, & \text{if } z_1(x) \leq a_1 \end{cases}$$

$$= \begin{cases} 0, & \text{if } z_1(x) \geq 5 \\ \frac{5 - (x_1 + x_2)}{5 - 3.5}, & \text{if } 3.5 \leq z_1(x) \leq 5 \\ \frac{(x_1 + x_2) - 2}{3.5 - 2}, & \text{if } 2 \leq z_1(x) \leq 3.5 \\ 0, & \text{if } z_1(x) \leq 2 \end{cases}$$

In the similar way,

$$\mu_2^I(x) = \begin{cases} 0, & \text{if } z_2(x) \geq 18 \\ \frac{18 - (3x_1 - 2x_2)}{18 - 9}, & \text{if } 9 \leq z_2(x) \leq 18 \\ \frac{(3x_1 - 2x_2) - (-0.01)}{9 - (-0.01)}, & \text{if } -0.01 \leq z_2(x) \leq 9 \\ 0, & \text{if } z_2(x) \leq -0.01 \end{cases}$$

If $\mu_1^I(x) = \max(\min(\frac{5 - (x_1 + x_2)}{5 - 3.5}, \frac{(x_1 + x_2) - 2}{3.5 - 2}), 0)$ and

$\mu_2^I(x) = \max(\min(\frac{18 - (3x_1 - 2x_2)}{18 - 9}, \frac{(3x_1 - 2x_2) - (-0.01)}{9 - (-0.01)}), 0)$ then $\mu_1^{I*}(1.71, 2.57)$ and $\mu_2^{I*}(3, 0)$.

The membership and non-membership functions are transformed by using first-order Taylor polynomial series

$$\mu_1^I(x) \cong \widehat{\mu_1^I}(x) = \mu_1^I(1.71, 2.57) + [(x_1 - 1.71)\frac{\partial \mu_1^I(1.71, 2.57)}{\partial x_1} + (x_2 - 2.57)\frac{\partial \mu_1^I(1.71, 2.57)}{\partial x_2}]$$

$$\mu_1^I(x) = 0.667x_1 - 0.667x_2 + 1.054 \quad (4.2)$$

$$\mu_2^I(x) \cong \widehat{\mu_2^I}(x) = \mu_2^I(3, 0) + [(x_1 - 3)\frac{\partial \mu_2^I(3, 0)}{\partial x_1} + (x_2 - 0)\frac{\partial \mu_2^I(3, 0)}{\partial x_2}]$$

$$\mu_2^I(x) = 0.333x_1 - 0.222x_2 + 0.001 \quad (4.3)$$

Then the objective of the FMOLPP is obtained by adding (4.2) and (4.3), that is

$p(x) = \widehat{\mu_1^I}(x) + \widehat{\mu_2^I}(x) = x_1 - 0.889x_2 + 1.055$

Subject to the constraints $2x_1 + x_2 \leq 6$

$x_1 + 4x_2 \leq 12$`

The problem is solved and the solution is obtained is as follows: $x_1 = 3$; $x_2 = 0$; $z_1(x) = 3$; $z_2(x) = 9$ and the membership values are $\mu_1 = 0.67$ and $\mu_2 = 1$.The membership fuction values show that both goals z_1 and z_2 are satisfied with 67% and 100% respectively for the obtained solution which is $x_1 = 3$; $x_2 = 0$.

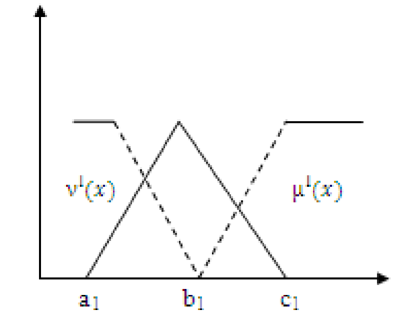

Figure 1:Membership and non-membership functions defined as intuitionistic triangular

The non-membership functions of the goals are as obtained as follows:

$$\nu_1{}^{I}(x)=\begin{cases}1\ ,\text{if } z_1(x)\geq c_1\\ \frac{z_1(x)-b_1}{c_1-b_1}\ ,\text{if } b_1\leq z_1(x)\leq c_1\\ \frac{b_1-z_1(x)}{b_1-a_1}\ ,\text{if } a_1\leq z_1(x)\leq b_1\\ 1\ ,\text{if } z_1(x)\leq a_1\end{cases}$$

$$=\begin{cases}1\ ,\ \text{if } z_1(x)\geq 5\\ \frac{(x_1+x_2)-3.5}{5-3.5}\ ,\text{if } 3.5\leq z_1(x)\leq 5\\ \frac{3.5-(x_1+x_2)}{3.5-2}\ ,\text{if } 2\leq z_1(x)\leq 3.5\\ 1\ ,\ \text{if } z_1(x)\leq 2\end{cases}$$

In the similar way,

$$\nu_2{}^{I}(x)=\begin{cases}1\ ,\ \text{if } z_2(x)\geq 18\\ \frac{(3x_1-2x_2)-9}{18-9}\ ,\text{if } 9\leq z_2(x)\leq 18\\ \frac{9-(3x_1-2x_2)}{9-(-0.01)}\ ,\text{if } -0.01\leq z_2(x)\leq 9\\ 1\ ,\ \text{if } z_2(x)\leq -0.01\end{cases}$$

If $\nu_1{}^{I}((x) = \max(\min(\frac{(x_1+x_2)-3.5}{5-3.5}, \frac{3.5-(x_1+x_2)}{3.5-2}), 1)$ and

$\nu_2{}^{I}(x) = \max(\min(\frac{(3x_1-2x_2)-9}{18-9}, \frac{9-(3x_1-2x_2)}{9-(-0.01)}), 1)$ then $\nu_1{}^{I*}(1.71,2.57)$ and $\nu_2{}^{I*}(3,0)$.

The non-membership functions are transformed by using first-order Taylor polynomial series

$$\nu_1{}^{I}(x)\cong\widehat{\nu_1}{}^{I}(x)=\nu_1{}^{I}(1.71,2.57)+[(x_1-1.71)\frac{\partial\nu_1{}^{I}(1.71,2.57)}{\partial x_1}+(x_2-2.57)\frac{\partial\nu_1{}^{I}(1.71,2.57)}{\partial x_2}]$$

$$\nu_1{}^{I}(x) = 0.667x_1-0.667x_2+1.574 \qquad (4.4)$$

$$\nu_2{}^{I}(x)\cong\widehat{\nu_2}{}^{I}(x)=\nu_2(3,0)+[(x_1-3)\frac{\partial\nu_2{}^{I}(3,0)}{\partial x_1}+(x_2-0)\frac{\partial\nu_2{}^{I}(3,0)}{\partial x_2}]$$

$$\nu_2{}^{I}(x) = 0.333x_1-0.222x_2+0.001 \qquad (4.5)$$

Then the objective of the FMOLPP is obtained by adding (4.4) and (4.5), that is

$q(x) = \widehat{\nu_1}{}^{I}(x) + \widehat{\nu_2}{}^{I}(x) = x_1-0.889x_2+1.575$

Subject to the constraints $2x_1+x_2\leq 6$

$x_1+4x_2\leq 12$

The problem is solved and the solution is obtained is as follows: $x_1=3$; $x_2 = 0$; $z_1(x)= 3$; $z_2(x)= 9$ and the non-membership values are $\nu_1 = 0.33$ and $\nu_2 = 0$.The non-membership function values show that both goals z_1 and z_2 are satisfied with 33% and 0% respectively for the obtained solution which is $x_1=3$; $x_2 = 0$.

b) The membership and non-membership functions were considered to be intuitionistic trapezoidal (see figure 2) when they depend on four scalar parameters (a_1,b_1,c_1,d_1).z_1 depends on four scalar parameters (-1, 2, 4, 4.5) when z_2 depends on four scalar parameters (7, 10, 11, 12). The membership and non-membership functions of the goals are as obtained as follows:

$$\mu_1{}^{I}(x)=\begin{cases}0\ ,\ \text{if } z_1(x)\geq d_1\\ \frac{d_1-z_1(x)}{d_1-c_1}\ ,\ \text{if } c_1\leq z_1(x)\leq d_1\\ 1\ ,\ \text{if } b_1\leq z_1(x)\leq c_1\\ \frac{z_1(x)-a_1}{b_1-a_1}\ ,\ \text{if } a_1\leq z_1(x)\leq b_1\\ 0\ ,\ \text{if } z_1(x)\leq a_1\end{cases}$$

$$=\begin{cases}0\ ,\ \text{if } z_1(x)\geq 4.5\\ \frac{4.5-(x_1+x_2)}{4.5-4}\ ,\ \text{if } 4\leq z_1(x)\leq 4.5\\ 1\ ,\text{if } 2\leq z_1(x)\leq 4\\ \frac{(x_1+x_2)-(-1)}{2-(-1)}\ ,\ \text{if } -1\leq z_1(x)\leq 2\\ 0\ ,\ \text{if } z_1(x)\leq -1\end{cases}$$

In the similar way,

$$\mu_2{}^{I}(x)=\begin{cases}0\ ,\ \text{if } z_1(x)\geq 12\\ \frac{12-(3x_1-2x_2)}{12-11}\ ,\ \text{if } 11\leq z_1(x)\leq 12\\ 1\ ,\ \text{if } 10\leq z_1(x)\leq 11\\ \frac{(3x_1-2x_2)-7}{10-7}\ ,\ \text{if } 7\leq z_1(x)\leq 10\\ 0\ ,\ \text{if } z_1(x)\leq 7\end{cases}$$

If $\mu_1{}^{I}(x) = \max(\min(\frac{4.5-(x_1+x_2)}{4.5-4}, \frac{(x_1+x_2)-(-1)}{2-(-1)}), 0)$ and

$\mu_2{}^{I}(x) = \max(\min(\frac{12-(3x_1-2x_2)}{12-11}, \frac{(3x_1-2x_2)-7}{10-7}), 0)$

then$\mu_1{}^{I*}(1.71,2.57)$ and $\mu_2{}^{I*}(3,0)$.

The membership and non-membership functions are transformed by using first-order Taylor polynomial series

$$\mu_1{}^{I}(x)\cong\widehat{\mu_1}{}^{I}(x)=\mu_1(1.71,2.57)+[(x_1-1.71)\frac{\partial\mu_1{}^{I}(1.71,2.57)}{\partial x_1}+(x_2-2.57)\frac{\partial\mu_1{}^{I}(1.71,2.57)}{\partial x_2}]$$

$$\mu_1{}^{I}(x) = 0.333x_1-2x_2+5.011 \qquad (4.6)$$

$$\mu_2{}^{I}(x)\cong\widehat{\mu_2}{}^{I}(x)=\mu_2(3,0)+[(x_1-3)\frac{\partial\mu_2{}^{I}(3,0)}{\partial x_1}+(x_2-0)\frac{\partial\mu_2{}^{I}(3,0)}{\partial x_2}]$$

$$\mu_2{}^{I}(x) = x_1-0.667x_2-2.333 \qquad (4.7)$$

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Then the objective of the FMOLPP is obtained by adding (4.6) and (4.7), that is

$p(x) = \widehat{\mu_1^I}(x) + \widehat{\mu_2^I}(x) = 1.333x_1 - 2.667x_2 + 2.678$

Subject to the constraints $2x_1 + x_2 \leq 6$

$$x_1 + 4x_2 \leq 12$$

The problem is solved and the solution is obtained is as follows: $x_1=3$; $x_2 = 0$; $z1(x)= 3$; $z2(x)= 9$ and the membership values are $\mu_1 = 1$ and $\mu_2 = 0.67$.The membership function values show that both goals z1 and z2 are satisfied with 100% and 67% respectively for the obtained solution which is $x_1=3$; $x_2 = 0$.

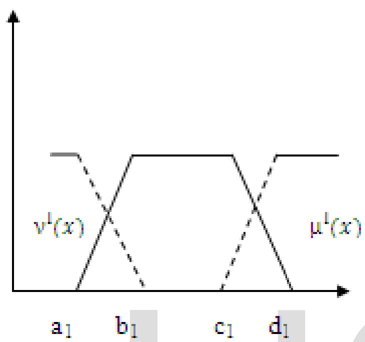

Figure 2: Membership and non-membership functions defined as intuitionistic trapezoidal

The non-membership functions of the goals are as obtained as follows:

$$\nu_1^I(x) = \begin{cases} 0 & , \text{ if } z_1(x) \geq d_1 \\ \frac{z_1(x) - c_1}{d_1 - c_1} & , \text{ if } c_1 \leq z_1(x) \leq d_1 \\ 1 & , \text{ if } b_1 \leq z_1(x) \leq c_1 \\ \frac{b_1 - z_1(x)}{b_1 - a_1} & , \text{ if } a_1 \leq z_1(x) \leq b_1 \\ 0 & , \text{ if } z_1(x) \leq a_1 \end{cases}$$

$$= \begin{cases} 1 & , \text{ if } z_1(x) \geq 4.5 \\ \frac{(x_1+x_2)-4}{4.5-4} & , \text{ if } 4 \leq z_1(x) \leq 4.5 \\ 0 & , \text{ if } 2 \leq z_1(x) \leq 4 \\ \frac{2-(x_1+x_2)}{2-(-1)} & , \text{ if} -1 \leq z_1(x) \leq 2 \\ 1 & , \text{ if } z_1(x) \leq -1 \end{cases}$$

In the similar way,

$$\nu_2^I(x) = \begin{cases} 1 & , \text{ if } z_1(x) \geq 12 \\ \frac{(3x_1-2x_2)-11}{12-11} & , \text{ if } 11 \leq z_1(x) \leq 12 \\ 0 & , \text{ if } 10 \leq z_1(x) \leq 11 \\ \frac{10-(3x_1-2x_2)}{10-7} & , \text{ if } 7 \leq z_1(x) \leq 10 \\ 1 & , \text{ if } z_1(x) \leq 7 \end{cases}$$

If $\nu_1^I(x) = \max\left(\min\left(\frac{(x_1+x_2)-4}{4.5-4}, \frac{2-(x_1+x_2)}{2-(-1)}\right), 1\right)$ and $\nu_2^I(x) = \max\left(\min\left(\frac{(3x_1-2x_2)-11}{12-11}, \frac{10-(3x_1-2x_2)}{10-7}\right), 1\right)$ then $\nu_1^{I*}(1.71,2.57)$ and $\nu_2^{I*}(3,0)$.

The non-membership functions are transformed by using first-order Taylor polynomial series

$$\nu_1^I(x) \cong \widehat{\nu_1^I}(x) = \nu_1^I(1.71,2.57) + \left[(x_1 - 1.71)\frac{\partial \nu_1^I(1.71,2.57)}{\partial x_1} + (x_2 - 2.57)\frac{\partial \nu_1^I(1.71,2.57)}{\partial x_2}\right]$$

$$\nu_1^I(x) = 2x_1 - 0.333x_2 - 2.564 \tag{4.8}$$

$$\nu_2^I(x) \cong \widehat{\nu_2^I}(x) = \nu_2^I(3,0) + \left[(x_1 - 3)\frac{\partial \nu_2^I(3,0)}{\partial x_1} + (x_2 - 0)\frac{\partial \nu_2^I(3,0)}{\partial x_2}\right]$$

$$\nu_2^I(x) = 3x_1 - 2x_2 - 9 \tag{4.9}$$

Then the objective of the FMOLPP is obtained by adding (4.8) and (4.9), that is

$q(x) = \widehat{\nu_1^I}(x) + \widehat{\nu_2^I}(x)) = x_1 - 0.889x_2 + 1.575$

Subject to the constraints $2x_1 + x_2 \leq 6$

$$x_1 + 4x_2 \leq 12$$

The problem is solved and the solution is obtained is as follows: $x_1=3$; $x_2 = 0$; $z_1(x)= 3$; $z_2(x)= 9$ and the non-membership values are $\nu_1 = 0$ and $\nu_2 = 0.33$.The non-membership function values show that both goals z1 and z2 are satisfied with 33% and 0% respectively for the obtained solution which is $x_1=3$; $x_2 = 0$.

5. Conclusion

In this paper, a powerful and robust method which is based on Taylor series is proposed to solve intuitionistic fuzzy multi-objective linear programming problems (IFMOLPP). Membership function and non-membership functions associated with each objective of the problem are transformed using Taylor series respectively.

Actually, the intuitionistic fuzzy multi-objective linear programming problem (IFMOLFP) is reduced to an equivalent multi-objective linear programming problem (MOLPP) by using the first order Taylor polynomial series. The obtained MOLLP problem is solved assuming that the weights of the objectives are equal. The proposed solution method will be applied to more variables and objectives. Thus, the performance of the proposed solution method was tested when the number of objectives and variables increased and when membership functions and non-membership functions were defined into distinct types. Then, the proposed solution method was applied to two numerical examples to test the effect on the performance of the proposed solution method with respect to distinct definitions of objectives and changes of the parameters. The proposed method facilitates computation to reduce the complexity in problem solving.

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